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1.INTRODUCTION

Apart from being influenced by static load far the most structures in building technology are influenced by dynamic load. Therefore, the resistance of a structure against dynamic effects - i.e. the properties of fatigue - has to be established.

Fatigue is often defined as fracture due to initiation and propagation of cracks as a result of repeated loads which do not themselves cause static fracture. Thus the resistance of a given structure against a static stress can be sufficient whereas a dynamic stress with a stress range which is much less than the static stress may lead to fracture. The stress range, $\Delta\sigma$, is defined as the difference between the maximum and the minimum stress in one load cycle, see also figure 1.1.

$$\Delta\sigma = \sigma_{max} - \sigma_{min} \tag{1.1}$$

where

σ_{max}	=	maximum stress in one load cycle [MPa]
σ_{min}	=	minimum stress in one load cycle [MPa]

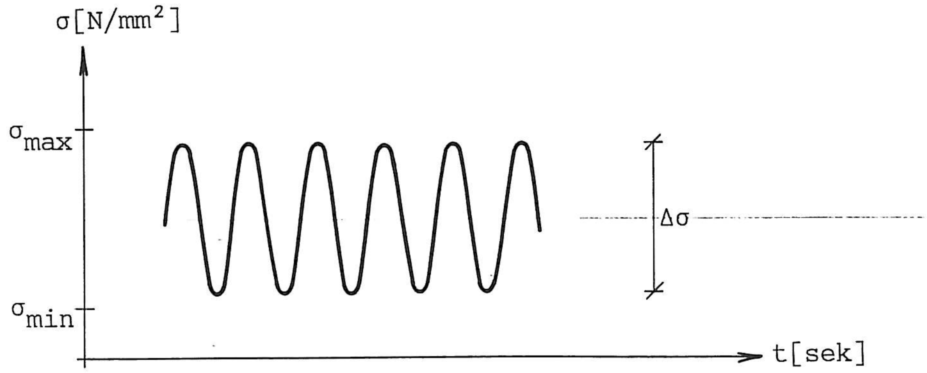


Figure 1.1: Specification of maximum and minimum stress in a load cycle.

Quantitative description of the properties of fatigue of a structure can be made in two principally different ways.

In one of the descriptions the properties of fatigue of a structure are characterized solely by knowledge of the total number of cycles to cause fracture, N_c, which for a given harmonic load are used to develop fatigue fracture. This description is typically represented by the so-called SN-curves (Wöhler-diagrams).

In the other description the properties of fatigue of a structure are characterized by the progress of the fatigue fracture in the form of crack growth. In this description the propagation phase of the fatigue fracture and the final state are dealt with. This description is represented by a crack propagation theory.

The resistance against fatigue of a structure can be established by the use of an SN-curve, which is illustrated in the light of a series of experiments carried out at different stress ranges, see figure 1.2. Each of the experiments is carried out at harmonic loading. For each of the experiments corresponding values of the number of cycles to cause fracture, N_c, and stress range $\Delta\sigma$ are registered. In this illustration it is traditional to denote the number of cycles to cause fracture N and the stress range S.

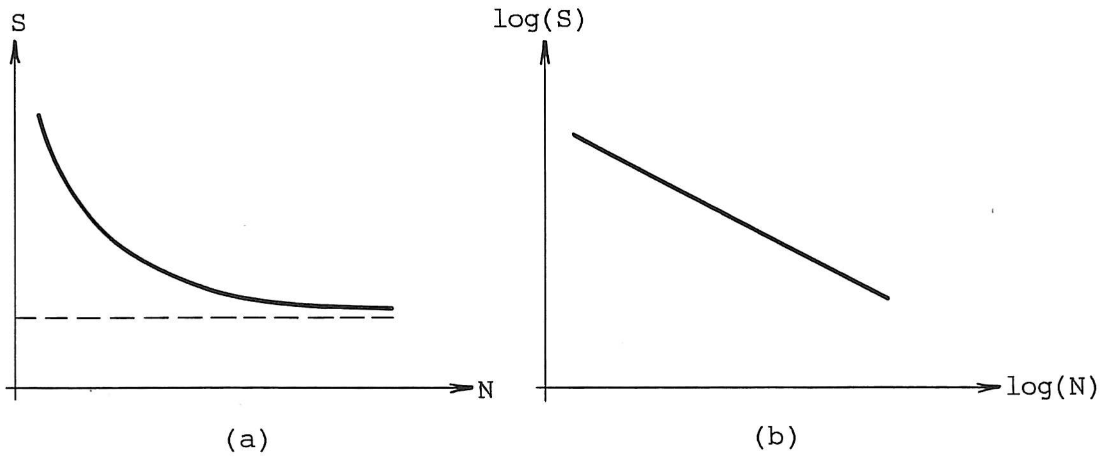


Figure 1.2: Typically SN-curve in arithmetric and logarithmic picture, respectively. $S = \Delta\sigma =$ stress range, $N = N_c =$ number of cycles to cause fracture.

It appears from figure 1.2b that a typical SN-curve forms a straight line in a double-logarithmic picture, i.e.

$$\log(\Delta\sigma) = \log(S) = k_1 \log(N) + k_2 \tag{1.2}$$

where

$$\begin{aligned} k_1 &= \text{slope of line} \\ k_2 &= \text{ordinate-intercept} \end{aligned}$$

If the number of cycles on the structure is known the maximal permissible stress range, $\Delta\sigma_c$, can be read. If the actual stress range on the structure is known then the maximal permissible number of cycles, N_c, can be found.

Thus, the permissible stress range can be established from tables if SN-data for similar structures exist. Otherwise it is necessary to carry out experiments.

As it is seen from figure 1.2a, a threshold value of the stress range appears so that stress ranges below this value do not cause fracture. Even though, the threshold value is not well defined it can, as a general rule, be assumed that the value has not been reached if the stress range does not cause fracture after 2-5 million cycles, [Gurney, T.R.;1979, p.14] or even more.

The uncertainty of the threshold value is due to the extensive use of time which is necessary for the determination, because the frequencies in the experiments are usually in the range 3-160Hz, [Gurney, T.R.;1979, p.11]. 8-12 corresponding SN-values are required for drawing an SN-curve.

Another description of the resistance against fatigue of a structure can, as mentioned earlier, be given by crack growth theories. The most frequently used crack growth theories are empirically based and express the resistance against crack propagation of a structure as

$$\frac{da}{dN} = f(\Delta K) \tag{1.3}$$

where

da	=	crack length increase [mm]
dN	=	increase in number of cycles
ΔK	=	$K_{max} - K_{min}$ = change in stress intensity factor in one load cycle. In the following ΔK is named stress intensity factor range [MPa$\sqrt{\text{m}}$]
K_{max}	=	maximum stress intensity factor in one load cycle [MPa$\sqrt{\text{m}}$]
K_{min}	=	minimum stress intensity factor in one load cycle [MPa$\sqrt{\text{m}}$]

The stress intensity factor K, which is a value from fracture mechanics, is defined by the stress field near a crack tip, c.f. [Hellan, K.;1985] and [Gansted, L.;1988]. In this way, this description assumes the existence of a crack.

The stress *distribution* in the neighbourhood of a crack is solely dependent on the position of the point considered in proportion to the crack tip, whereas the *magnitude* of the stresses depend on the applied load and the geometry of the structure and the crack, which is expressed by K.

The stress intensity factor K can in general be written as, see [Schijve, J.; 1979],

$$K = \sigma\sqrt{\pi a}F \tag{1.4}$$

where

$$\begin{array}{lcl} \sigma & = & \text{applied stress} \\ a & = & \text{crack length} \\ F & = & \text{factor which depends on geometry and loading} \end{array}$$

The dynamic effects can be divided into 3 categories, see figure 1.3, i.e.

(a) deterministic, cyclic, constant-amplitude loading

(b) deterministic, cyclic, variable-amplitude loading

(c) random loading

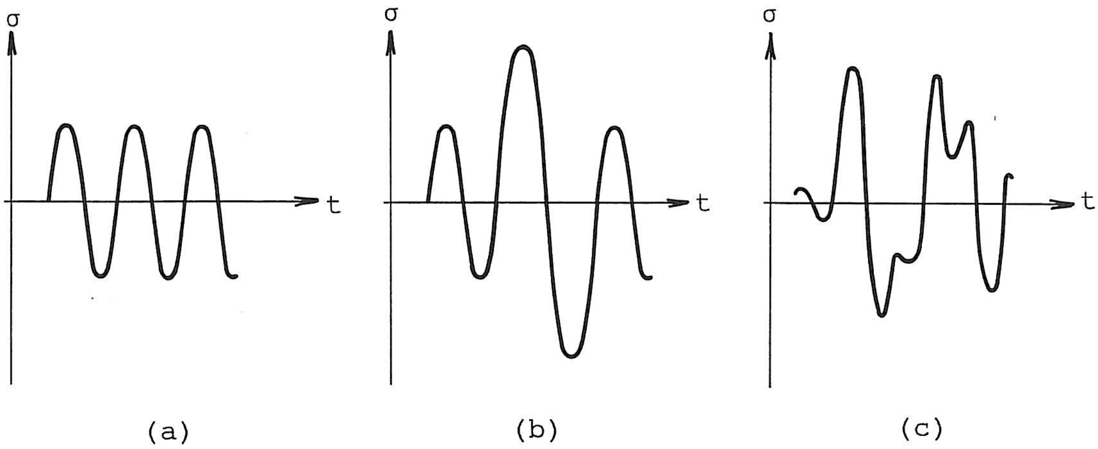


Figure 1.3: Different types of loading. (a) cyclic, constant-amplitude loading. (b) cyclic, variable-amplitude loading. (c) random loading.

For the categories (a) and (b) the progress of the fatigue fracture in a structure can be modelled by fracture mechanics. There is a number of empirical laws to express the crack growth as a function of the number of load cycles, see chapters 3 and 4. On the other hand, establishment of laws to describe the crack growth in structures influenced by random loading (c) is difficult , see chapter 5.

In addition to the type of loading the crack growth depends on

loading history

geometry of structure

material

boundary conditions

loading frequency

environment

In the following chapter 2 the ideas behind the crack growth theories are described, whereas the crack growth theories for the three loading categories (a)-(c) are dealt with in chapter 3-5.

2. CRACK PROPAGATION LAWS

As mentioned in chapter 1 it is possible to describe the fatigue process in two ways, namely the SN-curves and the crack growth theories. The SN-curves will not be further discussed, since only the crack growth theories give information on the fatigue progress itself, thereby making it possible to predict critical moments, e.g. increase of the crack growth rate.

To describe the crack growth in a structure subjected to fatigue a number of physically and/or empirically based crack growth theories are established. The basic ideas in these theories - stressing the parameters which have to be included - are described in the present chapter.

A crack growth theory must express the connection between the load process and the fatigue process.

The load process can be described by the applied stress range $\Delta\sigma$, see (1.1) and figure 1.1. The stress range $\Delta\sigma$ can be used both for deterministic, cyclic, constant-amplitude loads and at variable-amplitude loads because $\Delta\sigma$ is well defined. If, on the other hand, the load is random, the characterization of the load process depends on the method used for determining $\Delta\sigma$.

The fatigue process is characterized by the crack growth rate da/dN, see (1.3), and the total number of cycles to cause fracture, N_c. It should be mentioned that after a crack is initiated, about 75% of the number of cycles to cause fracture are spent on propagating the crack 25% of the final crack length a_f.

Thus, the crack growth theory must be able to describe this fatigue process.

A physically based crack growth model can be put forward using a dimensional analysis by which the physical problem, described by a dimensionally homogeneous equation, is reduced to an equation consisting of dimensionless products, cf. [Sabnis, G.M., Harris, H.G., White, R.N. and Mirza, M.S.;1983, p.33]. Dimensional analysis requires knowledge of the physical parameters assumed to have significant influence on the problem. In [Cherepanov, C.P. and Halmanov, H.;1972], from which the following is taken, an evaluation of the parameters that have influence on the crack growth rate da/dN and thus on the fatigue process is made.

The crack growth rate will depend on the load process represented by the stress intensity factor K, varying between K_{min} and K_{max}, and the number of cycles N. Furthermore, the properties of the material in the form of Young's modulus E, yield stress f_y, Poisson's ratio ν and the surface energy γ have influence. The surface energy is the energy used for the formation of a unit area new crack surface. Thus

$$\frac{da}{dN} = f(K_{max}, K_{min}, N, E, f_y, \nu, \gamma) \tag{2.1}$$

Information about the function f in (2.1) is established by an energy consideration. The crack energy calculated per unit thickness has to be equal to the total work per unit thickness performed by the process corresponding to the crack length increase δa, see also figure 2.1. Thus it is assumed that the energy does not vary in the direction of the thickness and the problem is thereby reduced to a two-dimensional problem.

$$2\gamma\delta a = \delta W + \delta A_p \tag{2.2}$$

where

δa	=	crack length increase [mm]
δW	=	crack work per unit thickness [Nm/mm]
δA_p	=	volume work per unit thickness [Nm/mm]

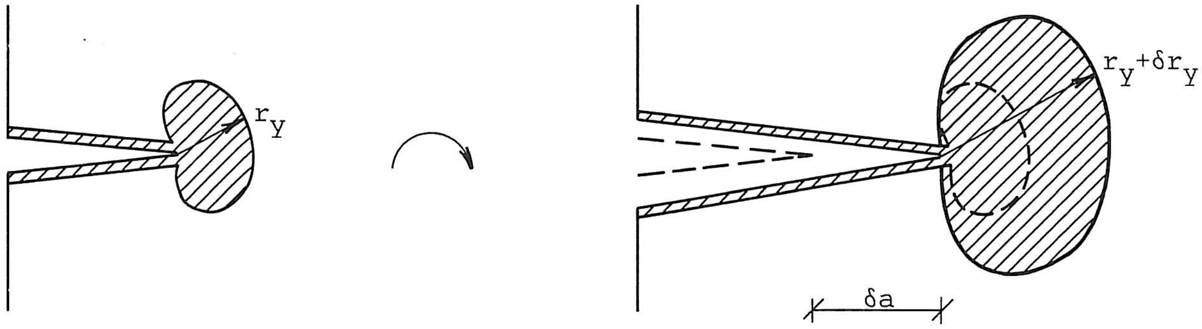


Figure 2.1: Schematic illustration of the crack and the plastic zone extension.

The crack work δW is the irreversible work done by the plastic zone as it performs a rigid-body motion in the direction of the crack extension. δW depends on the load parameter K, varying between K_{min} and K_{max}, and σ_0 describing the pre-loading history. Further the material parameters E, f_y and ν. Finally δW is dependent on the crack length increase δa. Dimensional analysis leads to

$$\delta W = 2\frac{K^2}{E}\delta a\ \alpha_2(\frac{f_y}{E}, \frac{\sigma_0}{E}, \nu) \tag{2.3}$$

where

α_2	=	dimensionless function

The independent variable in the dimensionless part containing δW is chosen as E because it is a measure of the resistance against crack propagation.

The volume work δA_p is the irreversible deformation work due to the increase in the plastic zone size during loading. δA_p depends on the same load and material parameters as δW. Further, δA_p is dependent on the change of load δK corresponding to the crack length increase δa. Dimensional analysis leads to

$$\delta A_p = 2\frac{K^3}{f_y^3}\delta K \; \alpha_3(\frac{f_y}{E}\frac{\sigma_0}{E}, \nu) \tag{2.4}$$

where

$\alpha_3 \quad = \quad$ dimensionless function

The independent variable in the dimensionless part containing δA_p is chosen as f_y because the size of the plastic zone r_y depends on the yielding properties of the material.

Insertion of (2.3) and (2.4) into (2.2) result in

$$\gamma = \frac{K^2}{E}\alpha_2(\frac{f_y}{E}, \frac{\sigma_0}{E}, \nu) + \frac{K^3}{f_y^3}\alpha_3(\frac{f_y}{E}, \frac{\sigma_0}{E}, \nu)\frac{\delta K}{\delta a}$$

which by reduction gives

$$\frac{\gamma E}{\alpha_2(\frac{f_y}{E}, \frac{\sigma_0}{E}, \nu)} = K^2 + \frac{\alpha_3(\frac{f_y}{E}, \frac{\sigma_0}{E}, \nu)}{\alpha_2(\frac{f_y}{E}, \frac{\sigma_0}{E}, \nu)} \frac{EK^3}{f_y^3} \frac{\delta K}{\delta a}$$

$\Downarrow$

$$K_c^2 - K^2 = \alpha_4(\frac{f_y}{E}, \frac{\sigma_0}{E}, \nu) \frac{EK^3}{f_y^3} \frac{\delta K}{\delta a}$$

$\Downarrow$

$$\frac{\delta a}{\delta K} = \alpha_4(\frac{f_y}{E}, \frac{\sigma_0}{E}, \nu) \frac{EK^3}{f_y^3(K_c^2 - K^2)} \tag{2.5}$$

where

$$\alpha_4(\frac{f_y}{E}, \frac{\sigma_0}{E}, \nu) = \frac{\alpha_3(\frac{f_y}{E}, \frac{\sigma_0}{E}, \nu)}{\alpha_2(\frac{f_y}{E}, \frac{\sigma_0}{E}, \nu)}$$

$$K_c^2 = \frac{\gamma E}{\alpha_2(\frac{f_y}{E}, \frac{\sigma_0}{E}, \nu)}$$

K_c is the critical stress intensity factor, see also section 3.1.

Integration of (2.5) over one load cycle gives

$$\Delta a = \alpha_4(\frac{f_y}{E}, \frac{\sigma_0}{E}, \nu)\ \frac{E}{f_y^3} \int_{K_{min}}^{K_{max}} \frac{K^3}{K_c^2 - K^2} dK$$

$\Downarrow$

$$\Delta a = \alpha_4(\frac{f_y}{E}, \frac{\sigma_0}{E}, \nu)\ \frac{E}{f_y^3}\ (-\frac{1}{2})\,[K_c^2\ ln\ \frac{K_c^2 - K_{max}^2}{K_c^2 - K_{min}^2} + (K_{max}^2 - K_{min}^2)]$$

$\Downarrow$

$$\Delta a = -\beta\ [\frac{K_{max}^2 - K_{min}^2}{K_c^2} + ln\ \frac{K_c^2 - K_{max}^2}{K_c^2 - K_{min}^2}] \qquad (2.6)$$

where

$$\beta = \alpha_4(\frac{f_y}{E}, \frac{\sigma_0}{E}, \nu)\ \frac{EK_c^2}{2f_y^3}$$

As continuous variables (2.6) becomes

$$\frac{da}{dN} = -\beta\ [\frac{K_{max}^2 - K_{min}^2}{K_c^2} + ln\ \frac{K_c^2 - K_{max}^2}{K_c^2 - K_{min}^2}] \qquad (2.7)$$

The critical stress intensity factor K_c is determined experimentally. The size of the parameter β depends on the type of load expressed by σ_0 which, as mentioned earlier, stands for the pre-loading history.

Only the number of cycles influence the following crack propagation if the loading is constant-amplitude. The order in which the cycles are applied at variable-amplitude loading and random loading is decisive, see chapter 4 and 5.

The pre-loading history can also be taken into account when calculating K_{max} and K_{min} whereby $\sigma_0 = 0$ and β becomes a constant.

Establishment of the empirical parameters, which enter the crack growth expressions, requires knowledge of the crack growth rate da/dN. Thus related values of a and N have to be transformed into da/dN. This differentiation can be made in several ways since the derivative of the function $a = f(N)$ is not uniquely established because, a and N are only known at discrete points. A correct equation for the crack growth rate will by integration reproduce the original a, N-data.

Instead of determining the crack growth parameters on the basis of da/dN [Ostergaard, D.F. and Hillberry, B.M.;1983] suggest that the parameters are determined directly from the a, N-data. This will correlate the crack growth rate equation directly with the raw data.

$$\int_0^N dN = N = \int_{a_0}^{a} \frac{1}{f} da \tag{2.8}$$

where f is given by (2.7), (3.1), (3.4), (5.1) or (5.2) perhaps in combination with (4.5)

The crack growth parameters which enter the right-hand side of (2.8) are estimated and then the equation is integrated numerically. This procedure is continued until the parameters describe the a, N-data as closely as possible.

Expression (2.7) forms the basis of the evaluation of the empirically based crack growth theories, see sections 3.2 and 4.3.

3. CRACK PROPAGATION UNDER DETERMINISTIC, CYCLIC, CONSTANT-AMPLITUDE LOADING

In the past 25 years several crack propagation laws to describe crack growth under constant-amplitude loading have been developed, see [Paris, P. and Erdogan, F.;1963]. Common to the laws is that they express the crack growth rate da/dN as a function of for instance the crack length a and the stress range $\Delta\sigma$, see figure 1.1.

The purpose of this chapter is to describe some of the crack propagation laws and their applications.

3.1 Empirical Crack Propagation Laws

In this section two of the most frequently used empirical crack propagation laws are described, i.e. Paris' law and Forman's law.

Paris' law, (3.1), expresses explicitly the connection between the loading process, which is applied to a given structure, and the process of fatigue in the structure. The loading process is described by the stress range $\Delta\sigma$ - see also chapter 2 - while the process of fatigue is characterized by the crack propagation rate da/dN and the number of cycles, N_c, to cause fracture. The process of fatigue can by Paris' law be described as a series of crack front extensions.

$$\frac{da}{dN} = C(\Delta K)^m \tag{3.1}$$

where

$$\begin{array}{lcl}
da & = & \text{increase in crack length [mm]} \\
dN & = & \text{increase in number of cycles} \\
C & = & \text{material constant } [\mathrm{mm}/(\mathrm{MPa}\sqrt{\mathrm{m}})^m] \\
m & = & \text{material constant} \\
\Delta K & = & K_{max} - K_{min} = \text{stress intensity factor range } [\mathrm{MPa}\sqrt{\mathrm{m}}] \\
K_{max} & = & \text{maximum stress intensity factor in one load cycle } [\mathrm{MPa}\sqrt{\mathrm{m}}] \\
K_{min} & = & \text{minimum stress intensity factor in one load cycle } [\mathrm{MPa}\sqrt{\mathrm{m}}]
\end{array}$$

The material constants C and m are correlated, cf. [Tanaka, S., Ichikawa, M. and Akito, S.;1981], but the correlation between them will depend on the physical units, [Yao, J.T.P., Kozin, F., Wen, Y.-K., Yang, J.-N., Schuëller, G.I. and Ditlevsen, O.;1986].

The stress intensity factor K, which is a quantity from fracture mechanics defined by the stress field near a crack tip, is among other parameters a function of the stress σ. Furthermore, K is dependent on the geometry of the crack and the structure.

The crack growth rate da/dN as a function of the stress intensity factor range ΔK will in a double-logarithmic illustration give a curve as shown in figure 3.1.

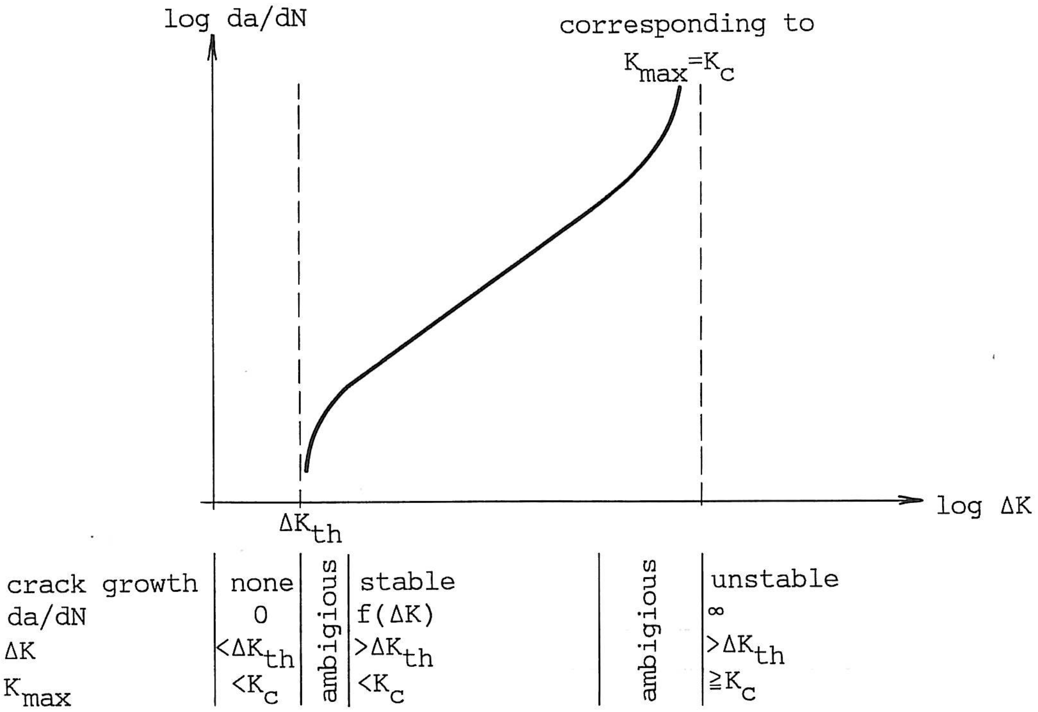


Figure 3.1: Typical $(\Delta K) - (da/dN)$ relationship in a double-logarithmic scale. ΔK_{th} = the threshold value of ΔK, K_c = the critical value of the stress intensity factor at which the final, uncontrolled fracture occurs. The dashed line to the right corresponds to $K_{max} = K_c$, where K_{max} is the maximum stress intensity factor in one load cycle.

It appears from figure 3.1 that Paris' law has only a complete validity in the interval $(\Delta K > \Delta K_{th} \wedge K_{max} < K_c)$, i.e. where the relationship is described by a straight line. The threshold value ΔK_{th} must be exceeded before crack growth occurs. When the maximum stress intensity factor K_{max} equals the critical value K_c the final, uncontrolled fracture takes place.

From expression (3.1) it is seen that da/dN does not depend upon the mean stress σ_m but only on the stress range $\Delta\sigma$. Regard to σ_m has been proposed by Forman, see [Forman, R.G., Kearnly, V.E. and Engle, R.M.;1967] and [Schijve, J.;1979].

As the maximum stress intensity factor equals the critical stress intensity factor K_c, the crack growth rate will, as shown in figure 3.1, assume infinitely values, i.e.

$$\lim_{K_{max} \to K_c} \frac{da}{dN} \to \infty \tag{3.2}$$

By inserting

$$K_{max} = \frac{\Delta K}{1 - R}$$

where

$$R = \frac{K_{min}}{K_{max}}$$

(3.2) becomes

$$\lim_{\Delta K \to (1-R)K_c} \frac{da}{dN} \to \infty \tag{3.3}$$

It is assumed that the crack growth rate has a form given by (3.1) and further, has a singularity given by (3.3). Forman has proposed

$$\frac{da}{dN} = \frac{C(\Delta K)^m}{(1 - R)K_c - \Delta K} \tag{3.4}$$

where

$$\begin{aligned} C &= \text{material constant} \\ m &= \text{material constant} \end{aligned}$$

and

K is given by (1.4)

Several other empirical crack growth laws exist. These will, however, not be discussed further because they are only variations of Paris' law.

Further, some energy-based crack growth laws exist, see [Kanazawa, T., Machida, S. and Itoga, K.;1975], [Zheng, X. and Hirt, M.A.;1983] and [Chand, S. and Girg, S.B.L.;1985]. Since the crack growth laws are not very frequently used, and since they are very different, they will not be further described in the present paper.

3.2 Evaluation of The Crack Propagation Laws

The purpose of this section os to evaluate the crack propagation laws, which are described in section 3.1, with regard to their applicability. The evaluation is performed by comparison with the crack propagation expression in chapter 2 and the experiments which are mentioned by different authors.

The validity of (2.7) has been investigated by [Cherepanov, G.P. and Halmanov, H.;1972] in which a serie of experiments make the basis of the investigation. The experiments, which are carried out with specimens made of aluminium alloys (2024-T3 and 7075-T6), show very good agreement with (2.7) at two different values of β.

The comparison between (2.7) and Paris' law (3.1) also show good agreement. The reason is that (2.7) can be reduced to (3.1) with $m = 4$ for $K_{min} = 0$. Experiments give $m = 3.6$ for the aluminium alloys mentioned above.

[Chow, C.L., Woo, C.W. and Chang, K.T.;1986] have made experiments with mild steel and aluminium alloys (7076 and 2024) among other materials. Even at different stress ratios, R, experiments show that the crack propagation rate is satisfactory described by (3.1). There is not complete agreement on the independence of the stress ratio. Thus, [Liaw, P.K., Leax, T.R., Fabis, T.R. and Donald, J.K.;1987] has observed that an increase of R results in an increase of the crack growth rate.

As described in section 3.1 Forman's equation (3.4) takes regard to the stress ratio. [Cherepanov, G.P. and Halmanov, H.;1972] has found that (3.4) gives a reasonable agreement with (2.7) for one set of the parameters (K_c, R, C and m). Other values of the parameters have not been used to compare (3.4) to (2.7).

Experiments carried out at different values of R with specimens made of 7075-T6 aluminium alloy show, according to [Chand, S. and Girg, S.B.L.;1985], that the crack growth is extremely well described by (3.4). Corresponding results appears from [Forman, R.G., Kearney, V.E. and Engle, R.M.;1967] for 7075-T6 and 2024-T3 aluminium alloys. Finally, the applicability of (3.4) can be seen in [Chow, C.L., Woo, C.W. and Chang, K.T.;1986], where it for both aluminium alloys (7076 and 2024) and mild steel has been observed agreement between experiments and (3.4).

Thus, it can be concluded that, both Paris' law (3.1) and Forman's equation (3.4) can describe the crack growth rate in a satisfactory way. But, after all, (3.4) gives the best description if different stress ratios appears. It appears, that the specimens in far the most experiments are made of aluminium alloy, but the conclusions can be transfered to mild steel.

4. CRACK PROPAGATION UNDER DETERMINISTIC, CYCLIC, VARIABLE-AMPLITUDE LOADING

As mentioned in chapter 1 far the most loads on structures in building technology are of a random character. In this chapter the effects of deterministic, cyclic, variable-amplitude loading, i.e. a loading in which every cycle is well defined, are investigated. These are investigated for the purpose of explaining the effects which can be registered at random loading, see chapter 5. Further, one of the crack growth laws which take account of variation of the amplitudes is described.

4.1 Loading Sequences and Their Effects

A possible grouping of deterministic, cyclic, variable-amplitude loading sequences is shown in figure 4.1 in which the most important variables are also given.

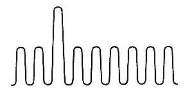
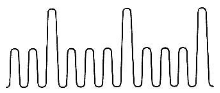
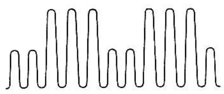
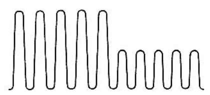
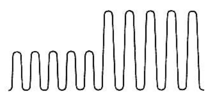
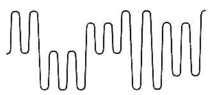
VARIABLE-AMPLITUDE LOADING		IMPORTANT VARIABLES	EXAMPLES
overloads	single overload	magnitude of overloads sequences in overload cycles	
	repeated overloads		
	blocks of overloads		
step loading	sequences of steps (high-low)	magnitude of steps	
	sequences of steps (low-high)		
programmed block loading	sequences of amplitudes	size of blocks distribution function of amplitudes	

Figure 4.1: Types of variable-amplitude loading and the most important variables. After [Schijve, J.;1976, p.7].

When a deterministic, cyclic, variable-amplitude loading acts on a structure the crack growth depends on the order in which the cycles are applied. The reason is that the crack growth in one cycle is a function of : the geometry of the crack before the cycle started, the state of the material at the crack tip and the magnitude of the load cycle.

In the following some of the effects which can be observed if a structure in modus I is subjected to overloads are described. An overload is a cycle whose numerical value and range exceed the most common cycles. Modus I corresponds to an opening mode in which the load acts orthogonally to the direction of the crack propagation, see also [Hellan, K.;1985] and [Gansted, L.;1988].

The most simple case is a single overload. The overload sequences shown in figure 4.2 are considered.

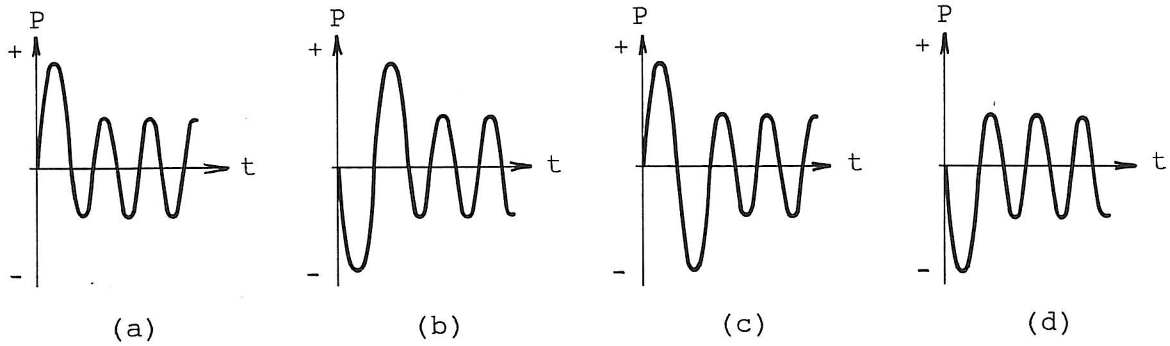


Figure 4.2: Overload sequences. (a) Tensile overload. (b) Compressive-tensile overload. (c) Tensile-compressive overload. (d) Compressive overload. After [Fuchs, H.O. and Stephens, R.I.;1980, p. 194].

The loading sequence shown in figure 4.2(a) causes a delay in the crack growth (retardation) while the situation in figure 4.2(d) results in an acceleration of the crack growth. The effect of acceleration is less pronounced than the effect of retardation, cf. [Schijve, J.;1976]. The immediate cause for this deviation seems to be different material properties in tension and compression. A more plausible explanation is that the in the latter case the material is compressed, and thus the material is accumulated within a given space. This is considered to be more difficult than separation of the material (tension).

Likewise, if a compressive overload is followed by a correspondingly tensile overload - figure 4.2(b) - a retardation of the crack growth is obtained. However, it is not as pronounced as the purely tensile overload, figure 4.2 (a). A small retardation is obtained if a compressive overload succeeds a tensile overload, figure 4.2(c). This has also been observed by [Porter, T.R.;1972].

The crack growth progress in above-mentioned cases (a)-(d) is shown in figure 4.3.

Several experiments show that for a given value of low-load, the higher the value of high-load, the greater is the delay in crack propagation. This has been observed by [Hudson, C.M. and Raju, K.N.;1970], [Porter, T.R.;1972] (both on aluminum alloy 7075-T6), [Probst, E.P. and Hillberry, B.M.;1974] (aluminum alloy 2024-T3) and [Rice, R.C. and Stephens, R.I.;1973] (low strength steel).

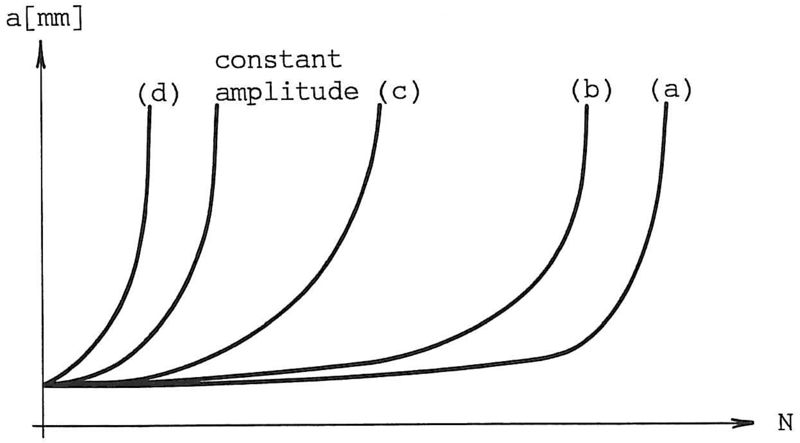


Figure 4.3: Crack growth curves for loading as shown in figure 4.2 (a)-(d). After [Fuchs, H.O. and Stephens, R.I.;1980].

Occasionally it is possible to observe a delayed retardation of the crack growth as a consequence of a tensile overload. This form of retardation appears primarily when the overload is much greater than the maximum load in the other cycles in the loading sequence. Thus, an acceleration is initially obtained before the retardation occurs, see [Schijve, J.;1976] and [Rice, R.C. and Stephens, R.I.;1973] and figure 4.4.

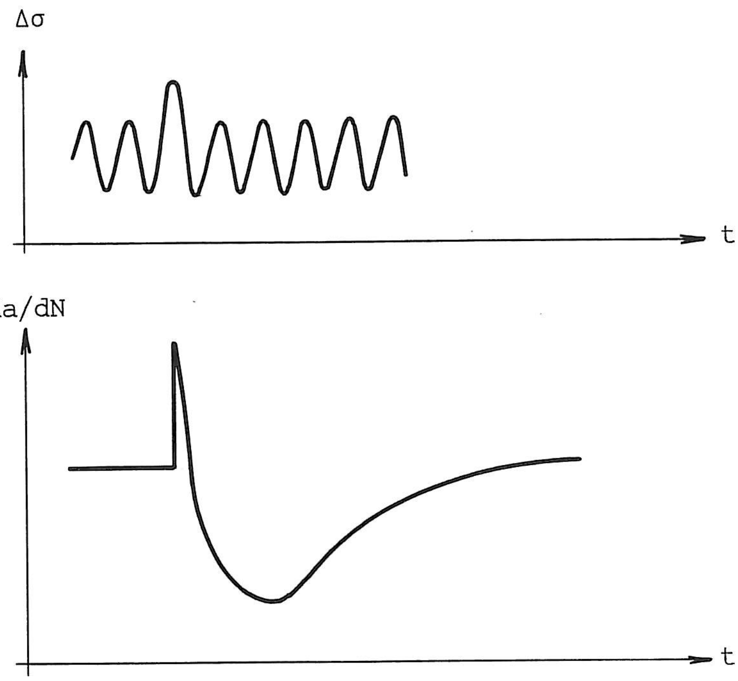


Figure 4.4: Illustration of the crack growth rates before and after a single tensile overload, which causes delayed retardation. After [Corbly, D.M. and Packman, P.F.;1973].

From figure 4.4 it is seen that the crack growth behavior following a single tensile overload consists of a delayed retardation period (momentary acceleration and then retardation) followed by an increasing crack growth rate until the original crack growth rate is obtained.

By repeated overloads or blocks of overloads the same effects as mentioned above are obtained but to a higher degree. [Porter, T.R.;1972] found that the more cycles at small amplitude between the repeated overloads, the longer the retardation. This has also been observed by [Rice, R.C. and Stephens, R.I.;1973].

Furthermore, the retardation is increased with increasing numbers of overload cycles up to a limit. If only 1 overload is introduced 25% of the maximum retardation will be obtained whereas 10 overloads in one block result in 50% of the maximum retardation, see [Himmelein, M.K.;1974], [Corbly, D.M. and Packman, P.F.;1973] and [Hudson, C.M. and Raju, K.N.;1970].

Complete crack arrest at the small amplitude level for overload/small amplitude ratios greater than 1.5-2.0 has been observed when more than one overload is introduced.

The possible causes of the interaction effects by variable-amplitude loading are described in the following.

When an overload succeeds a sequence of small-amplitude cycles this overload gives rise to a plastic zone of larger extension than the plastic zone originating from one of the preceding small-amplitude cycles. At the moment the overload decreases from its maximum value to zero - i.e., when no external load is acting on the crack tip - residual stresses will arise in the plastic zone.

The sign of these residual stresses depends on the sign of the external load that caused the plastic zone, i.e. the sign of the overload. A tensile overload will lead to compressive residual stresses and vice versa.

When subsequently the amplitudes of the following cycles reassume their former value the effect from these on the crack growth is influenced by the residual stresses. Likewise, a great deal of the following small-amplitude cycles will be spent surmounting the zone of residual stresses. Not until the residual stresses have been eliminated at the crack tip will the crack growth rate assume its previous value.

Due to the compressive residual stresses the crack is not completely open when loaded. This is because the plastic deformations result in contact between the fracture surfaces before complete unloading has taken place. Crack closure is said to occur, cf. [Schijve, J.;1979] and [Corbly, D.M. and Packman, P.F.;1973]. The phenomenon of crack closure is further described in [Gansted, L.;1988].

The crack is thus closing at a stress which exceeds the stress corresponding to complete unstressing. The effective stress range $\Delta\sigma_{eff}$ is hereby reduced

$$\Delta\sigma_{eff} = \sigma_{max} - \sigma_{cl} \tag{4.1}$$

where

σ_{max}	=	maximum stress in a load cycle [MPa]
σ_{cl}	=	crack closure stress = stress at which the crack is closed. [MPa]

Crack propagation only takes place when the crack is fully opened, and therefore, the reduced effective stress gives rise to a smaller crack growth. Retardation is said to occur.

Further crack tip blunting causes a reduction of the stress concentration resulting in smaller crack growth, cf. [Druce, S.G., Beevers, C.J. and Walker, E.F.;1979]. Crack tip blunting is due to high tensile stresses.

The tensile residual stresses give rise to an increased effective stress whereby the crack growth is accelerated.

The crack closure stress σ_{cl} will vary at variable-amplitude loading. The variation of σ_{cl} in a loading sequence consisting of cyclic, deterministic, constant-amplitude loading and one tensile overload appear from figure 4.5.

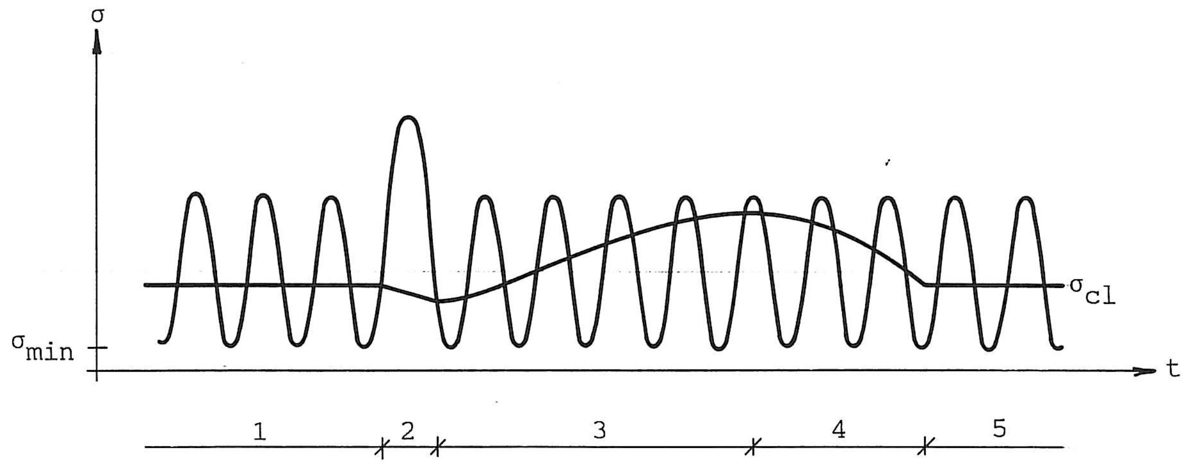


Figure 4.5: Variation in crack closure stress when one overload is introduced. 1: $\sigma_{cl} \geq \sigma_{min}$ 2: Decrease in σ_{cl} 3: $\sigma_{cl} > \sigma_{min}$, σ_{cl} increasing 4: $\sigma_{cl} > \sigma_{min}$, σ_{cl} decreasing 5: $\sigma_{cl} \geq \sigma_{min}$.

For constant-amplitude loading $\sigma_{cl} \geq \sigma_{min}$. After introducing the tensile overload a decrease in σ_{cl} appears which causes an accelerated crack growth. After this σ_{cl} is increasing and when $\sigma_{cl} > \sigma_{min}$ the effective stress range $\Delta\sigma_{eff}$ is reduced, i.e. retardation occurs. At some moment another decrease in σ_{cl} occurs and so $\Delta\sigma_{eff}$ is increased. When σ_{cl} reassumes the value for constant-amplitude loading the retardation is said to be ended.

This variation in σ_{cl} is among others observed by [Schijve, J.;1979] and [Fuchs, H.O. and Stephens, R.I.;1980].

4.2 Wheelers Crack Propagation Model

Several crack propagation laws exist which take account of retardation due to overloads. The most important is the Wheeler-model.

The model seeks correlating crack propagation due to variable-amplitude loading with crack propagation due to constant-amplitude loading.

The Wheeler-model which gives an empirical description of the crack growth, see [Wheeler, O.E.;1972], takes its origin in the crack growth-theoretical connection between the crack growth rate da/dN due to constant-amplitude loading and the stress intensity factor range ΔK.

$$\frac{da}{dN} = f(\Delta K) \tag{4.2}$$

From (4.2) it is seen that the crack length after N cycles of constant-amplitude can be approximately expressed as

$$a_N = a_0 + \sum_{i=1}^{N} f(\Delta K_i) \tag{4.3}$$

where

a_0	=	initial crack length [mm]
ΔK_i	=	stress intensity factor range in the i'th cycle [MPa$\sqrt{\text{m}}$]

Wheeler proposes (4.2) corrected for variable-amplitude loading by introducing an empirical retardation parameter $C_{ret,i}$ for the i'th load cycle after a tensile overload.

$$C_{ret,i} = \begin{cases} [\frac{r_{yi}}{a_{0L}+r_{0L}-a_i}]^d & \text{for } a_i + r_{yi} < a_{0L} + r_{0L} \\ 1 & \text{for } a_i + r_{yi} > a_{0L} + r_{0L} \end{cases} \tag{4.4}$$

where

r_{yi}	=	extension of the plastic zone at the crack tip in the i'th load cycle [mm]
r_{0L}	=	extension of the plastic zone due to a previous overload [mm]
a_{0L}	=	crack length at the time when overload is introduced [mm]
a_i	=	crack length to an arbitrary time after introducing the overload [mm]
d	=	empirical coefficient

The meaning of the symbols is also shown in figure 4.6.

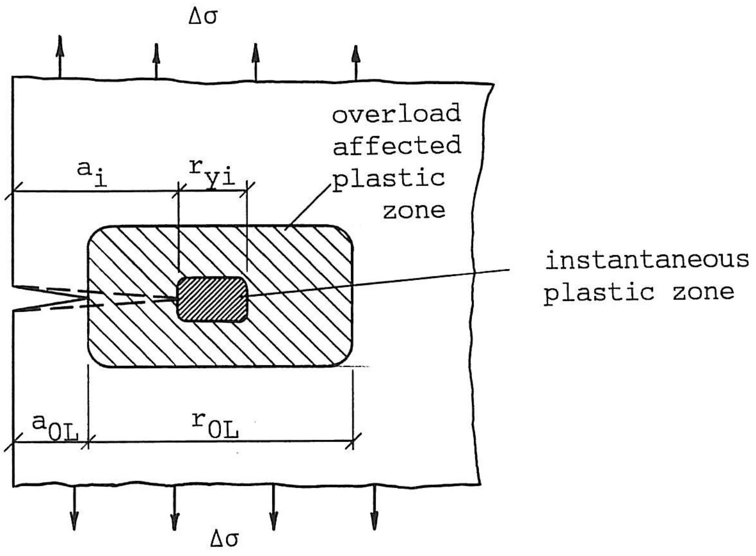


Figure 4.6: Schematic outline of plastic zones at the crack tip due to load cycles of large and small amplitude, respectively.

By introducing (4.4) in (4.3) the Wheeler crack length is

$$a_N = a_0 + \sum_{i=1}^{N} C_{ret,i} f(\Delta K_i) \tag{4.5}$$

where

f for example is given by Paris' law (3.1) or Forman's equation (3.4)

The crack growth in the i'th load cycle is retarded if the plastic zone due to this cycle is situated inside the plastic zone due to the overload. When the actual cycle is causing a plastic zone which exceeds the extension of the plastic zone due to the large load cycle the effect of retardation is assumed to be ended. For $d = 0$ no retardation of the crack growth occurs.

Characteristic of the Wheeler-model is that it does not give any possibility for taking accelerated growth in to consideration. Therefore, the model cannot describe delayed retardation.

4.3 Evaluation of Wheelers Model

The pupose of this section is to evaluate Wheelers model (4.5), which describes crack growth under deterministic, cyclic variable-amplitude loading, see section 4.2. The evaluation is primarily performed by comparison with the experiments which are mentioned by different authors.

The validity of (2.7) was proven in section 3.2. The parameter σ_0, which stands for the pre-loading history, is changed and thus β is changed, when variable-amplitude loading is introduced. A direct comparison between (4.5) and (2.7) has not been performed.

Wheelers model gives a good describtion of the retardation due to deterministic, cyclic loading in which one or more tensile overloads are introduced, cf. [Corbly, D.M. and Packman, P.F.;1973] and [Wheeler, O.E.;1972].

The effects of acceleration can not be described by Wheelers model, and therefore, it is not possible to take regard to delayed retardation. That is, if a great overload appears, Wheelers model results in a poorer description of the crack growth rate.

Thus, it can be concluded that in many cases Wheelers model gives a good description of the crack growth.

5. CRACK PROPAGATION UNDER RANDOM LOADING

As mentioned in chapter 1, the dynamic load on a structure are often of random character. The purpose of this chapter is to describe how the random load process can be characterized and which parameters that have to be considered when describing the fatigue process. Further, some crack growth models are described and evaluated.

5.1 Introduction

Unlike the deterministic loading, the random loading can not be defined by a mathematical expression. Therfore, the random loading is often modelled as Gaussian stochastic processes characterized by the mean and the variance, cf. [Madsen, H.O.;1982] and [Schijve, J.;1979]. Further, it is often assumed that the process is stationary, i.e. the statistical properties are independent of time. Both Gaussian and stationary stochastic processes are further described in e.g. [Parzen, E.;1962].

Different realizations of the loading history can be performed by simulation if the distribution of the stochastic process is known, see figure 5.1. Then, the crack growth rate can be expressed, because the random loading is decomposed into a collection of discrete load cycles.

These load cycles have to be defined and counted. Several counting methods exist of which the Rainflow Counting Method and the Range Pair Counting Method are the two most common.

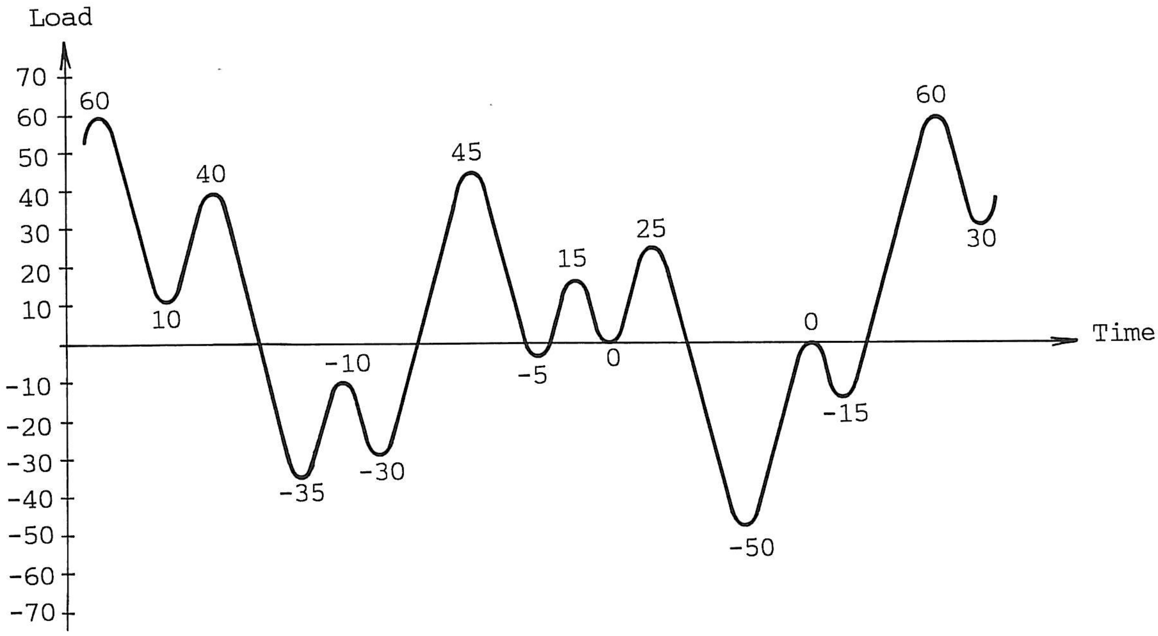


Figure 5.1: Realization of a stochastic process.

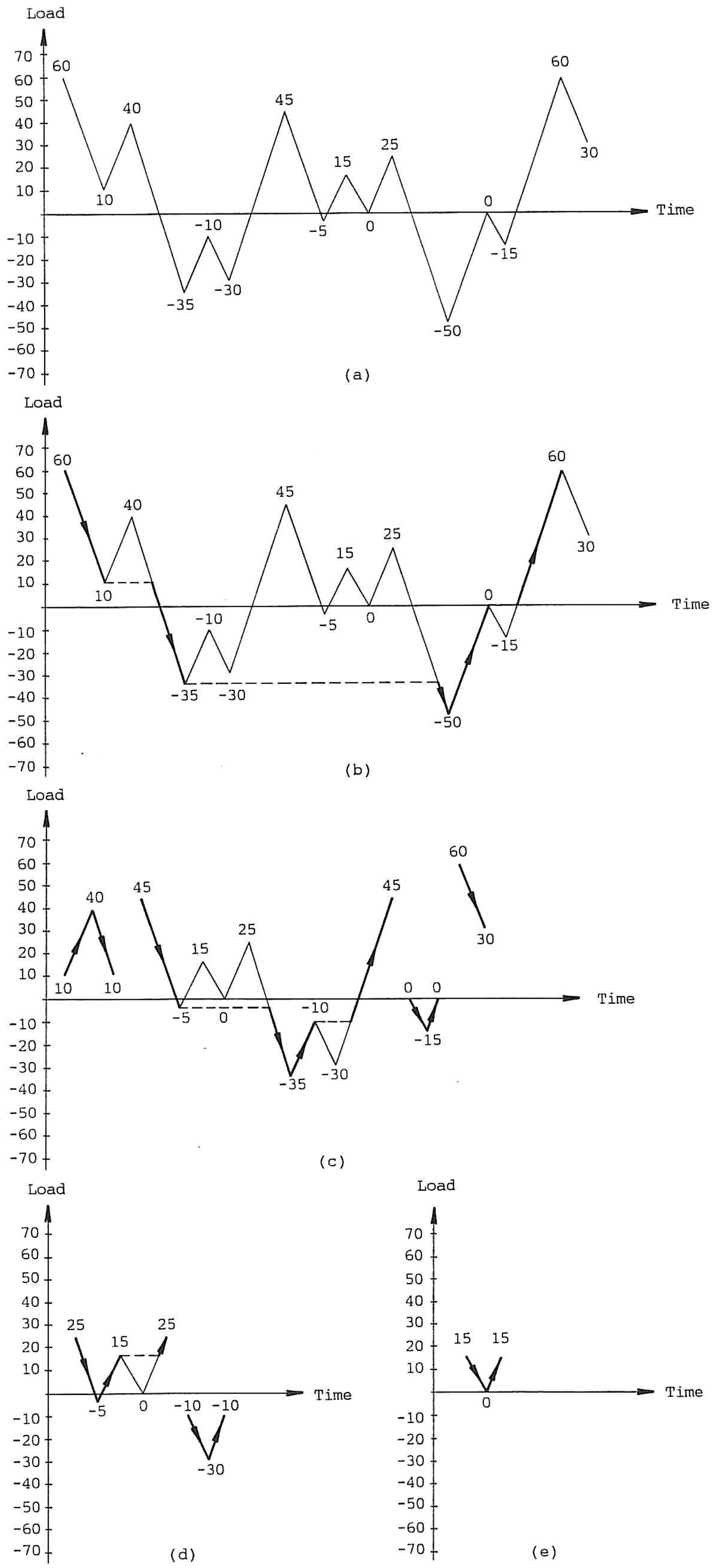


Figure 5.2: The principle in the Rainflow Counting Method.

The procedure in the Rainflow Counting Method appears from the following and figure 5.2. The text in the brackets refers to figure 5.2.

1. The realization of the stochastic process is replaced by a sawtooth pattern with the magnitude of the peaks and troughs unchanged. (Figure 5.1 → figure 5.2a).
2. The loading is rearranged to start with the highest peak, (Figure 5.2b: 60).
3. Start from the highest peak and go down to the next reversal, (60 → 10). Proceed horizontally to the next downward range and go down to the next reversal, (10 → -35) and (-35 → -50). If there is no range going down from the level of the trough at which was stopped, proceed upward to the next reversal, (-50 → 0).
4. Repeat the same procedure upward instead of downward, (0 → 60) and continue these steps to the end.
5. Repeat the procedure 2-4 for all parts that were not used in previous procedures. (Figure 5.2c: 60 → 30, 45 → -5 → -35, -35 → -10 → 45, 0 → -15, -15 → 0, 10 → 40, 40 → 10, figure 5.2d: 25 → -5, -5 → 15 → 25, -10 → -30, -30 → -10 and figure 5.2e: 15 → 0 → 15).

Thus, it is assumed that the numerical greatest peak value is positive. If this is not the case, start from the smallest trough and go upward instead.

Each complete path, e.g. (60 → 10 → -35), is regarded as a half cycle (simple range). The half cycles are counted and the result for the realization in figure 5.1 and figure 5.2 is given in tabel 5.3.

Range	Positive half-cycles	Negative half-cycles	Total cycles
110	-50 → 0 → 60	60 → 10 → -35 → -50	1
80	-35 → -10 → 45	45 → -5 → -35	1
30	10 → 40, -5 → 15 → 25	40 → 10, 60 → 30, 25 → -5	2 $\frac{1}{2}$
20	-30 → -10	-10 → -30	1
15	-15 → 0, 0 → 15	0 → -15, 15 → 0	2

Tabel 5.3: Rainflow Counting corresponding to figure 5.1 and figure 5.2.

One of the disadvantages using the Rainflow Counting Method is that information on the stress ranges which have actually occurred is discarded. Further, stress ranges of greater size are introduced, compare tabel 5.3 and tabel 5.4.

In the Range Pair Counting Method, a range is defined as the part of the realization between two adjacent points of reversal. Both the positive (ascending) and the negative (descending) ranges are counted. The cycles are formed by pairing positve and negative ranges of the same size. A more simple method is to count all the ranges, positive as well as negative, and then the total number of ranges of same size is divided by two in order to get the number of cycles of the corresponding size. The results for the realization in figure 5.1 is given in tabel 5.4.

Range	Positive ranges	Negative ranges	Total cycles
75	-30 → 45, -15 → 60	40 → -35, 25 → -50	2
50	-50 → 0	60 → 10, 45 → -5	$1\frac{1}{2}$
30	10 → 40	60 → 3	1
25	-35 → -10, 0 → 25		1
20	-5 → 15	-10 → -30	1
15		15 → 0, 0 → -15	1

Tabel 5.4: Range Pair Counting corresponding to figure 5.1.

In this method, information on the stress range which have actually occurred is retained, but information on the level at which they have occurred (mean stress and peak stress of each range) gets lost.

A disadvantage using the Rainflow Counting Method or the Range Pair Counting Method is that the sequence of the cycles gets lost. As described in chapter 4, the order in which the load cycles are applied on a structure have a descisive influence on the crack growth progress. This is due to the interaction effects (retardation and acceleration).

The crack growth rate da/dN can, if the loading history is of a relativ simple random nature, be described directly as a random process determined by the loading history, cf. [Arone, R.;1986].

If the case is not as above, the crack growth rate has to be described by a different random process. [Jacoby, G.H. and Nowack, H.;1972] have by analysis of test data concluded that both the Weibull distribution and the Log-normal distribution are able to describe the crack growth rate.

Determination of the load process is not the only parameter, which influence the fatigue crack growth in a structure influenced by random loading.

Only if one or more cracks are initiated and propagated, fatigue can appear in a structure. Thus, it has to be established where the cracks will appear and which size they will have. This subject will not be further discussed in this paper, but the problem has been investigated by [Berens, A.P. and Hovey, P.W.; 1983] and [Trantia, G.G. and Johnson, C.A.;1983].

Fatigue and reliability are closely related. Thus, a fatigue reliability analysis consists of estimating the probability of survival (non-failure and the probability of safe operation (strength greater than a certain limit) when a structure is subjected to a random loading. The relation between fatigue and reliability is investigated by [Talreja, R.;1979a] and [Talreja, R.;1979b]. This subject is nor further discussed in this paper.

Finally, a proper fatigue crack growth model has to be selected so the fatigue in similar structures influenced by random loading can be described. Some of the existing models are investigated in section 5.2.

5.2 Fatigue Crack Growth Models

In this section different crack growth models are described. The first and second are based on crack growth models used in constant-amplitude loading.

The most common is to use the root mean square value of the stress intensity factor, ΔK_{rms}, for example in combinaton with Paris' law (3.1).

$$\frac{da}{dN} = C(\Delta K_{rms})^m \tag{5.1}$$

where

da	=	increase in crack length [mm]
dN	=	increase in number of cycles
C	=	material constant [mm/(MPa$\sqrt{\text{m}}$)^m]
m	=	material constant
ΔK_{rms}	=	$[\frac{1}{N}\sum_{i=1}^{N}(\Delta K_i)^2]^{\frac{1}{2}}$ [MPa$\sqrt{\text{m}}$]
N	=	number of cycles
ΔK_i	=	stress intensity factor range in the i'th cycle [MPa$\sqrt{\text{m}}$]

This method is used by several authors, e.g. [Barsom, J.M.;1973] and [Alawi, H.;1986], and it gives a good describtion of the crack growth rate if dN is chosen small.

[Hudson, C.M.;1981] used ΔK_{rms} in the Forman equation (3.4), i.e.

$$\frac{da}{dN} = \frac{C(\Delta K_{rms})^m}{(1 - R_{rms})K_c - \Delta K_{rms}} \tag{5.2}$$

where

K_c	=	critical stress intensity factor [MPa$\sqrt{\text{m}}$]
ΔK_{rms}	=	$K_{max,rms} - K_{min,rms}$ [MPa$\sqrt{\text{m}}$] = root mean square value of stress intensity factor
R_{rms}	=	$K_{min,rms}/K_{max,rms}$ = root mean square stress intensity factor ratio
$K_{max,rms}$	=	$[\frac{1}{N}\sum_{i=1}^{N}(K_{max})^2]^{\frac{1}{2}}$ [[MPa$\sqrt{\text{m}}$]
$K_{min,rms}$	=	$[\frac{1}{N}\sum_{i=1}^{N}(K_{min})^2]^{\frac{1}{2}}$ [MPa$\sqrt{\text{m}}$]

(5.2) gives a good description of the crack growth rate at different stress ratios, c.f. [Hudson, C.M.;1981], see also section 3.2.

Neither (5.1) nor (5.2) can describe the interaction effects because they do not distinguish between tensile and compressive overloads. All attempts to use constant-amplitude models as a basis of a model describing random fatigue appears to fail.

Several other attempts have been made. For example [Chakrabarti, A.K.;1980] uses a dislocation model in which it is assumed that, the crack only grow when the dislocation distribution becomes critical. The critical dislocation distribution is difficult to determine. This model will not be further discussed in this paper.

Until now, no adequate model has been purposed but, [Bogdanoff, J.L. and Kozin, F.;1985] have developed a numerical model based on Markov Chain theory and the state of damage. It looks like, this model might be useful and it will be further discussed in coming papers.

6. CONCLUSION

The main purpose of this paper was to describe and evaluate the existing crack growth expressions.

In chapter 3 it was found that the empirical Paris' law (3.1) gives a good description of crack growth in a structure subjected to constant-amplitude loading. A disadvantage is that the material parameters C and m must be determined experimentally and that the variance - especially for C is great.

It was also found that the Forman equation (3.4) is useful when different stress ratios is introduced, but still it is only valid for constant-amplitude loading.

If the structure is subjected to variable-amplitude loading the crack growth model has to take into account the interaction effects due to overloads. The effects are retardation, acceleration and delayed acceleration.

The Wheeler model (4.5) seems to give a reasonable description of the crack growth rate if the overloads are tensile. But, if the overloads are of compressive character, the Wheeler model fails.

Crack growth under random loading was dealt with in chapter 5. On the basis of chapter 4 and chapter 5, it can be concluded that a good fatigue crack growth model must be able to describe:

- retardation (residual compressive stresses)
- acceleration (residual tensile stresses)
- delayed retardation (acceleration + retardation)
- crack closure
- crack growth due to random material properties
- the sequence in the load cycles

As it was mentioned in chapter 5, no adequate model exists. Either an analytical or a numerical model, which can be verified experimentally, and which fulfills the demands above, is still an unsolved problem in fatigue analysis.

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SYMBOLS

a	=	crack length
a_0	=	initial crack length
a_{0L}	=	crack length at the time when overload is introduced
a_c	=	critical crack length
a_f	=	final crack length
a_i	=	crack length to an arbitrary time after introducing an overload
a_N	=	crack length corresponding to N
$\Delta a, \delta a$	=	crack length increase
δA_p	=	volume work per unit thickness
C	=	material constant
C_{ret}	=	retardation parameter
d	=	empirical coefficient
da	=	crack length increase
dN	=	increase in number of cycles
E	=	Young's modulus
f	=	function
f_y	=	yield stress
k_i	=	constant, $i = 1, 2$
K	=	stress intensity factor
K_c	=	critical stress intensity factor
K_{max}	=	maximum stress intensity factor
$K_{max,rms}$	=	root mean square of the maximum stress intensity factor
K_{min}	=	minimum stress intensity factor
$K_{min,rms}$	=	root mean square of the minimum stress intensity factor
ΔK	=	stress intensity factor range
ΔK_i	=	stress intensity factor range in the i'th load cycle
ΔK_{rms}	=	root mean square value of stress intensity factor range
ΔK_{th}	=	threshold value of ΔK
m	=	material constant
N	=	number of cycles
N_c	=	number of cycles to cause fracture
r_{0L}	=	plastic zone size due to a previous overload
r_y	=	plastic zone size
r_{yi}	=	plastic zone size corresponding to the i'th load cycle
R	=	K_{min}/K_{max} = stress intensity factor ratio
R_{rms}	=	$K_{min,rms}/K_{max,rms}$ = root mean square stress intensity factor ratio
S	=	stress range
δW	=	crack work per unit thickness
α_i	=	dimensionless function, $i = 2, 3, 4$
β	=	function
γ	=	surface energy

ν	=	Poisson's ratio
σ	=	stress
σ_0	=	stress which states for pre-loading history
σ_c	=	critical stress
σ_{cl}	=	crack closure stress
σ_m	=	mean stress
σ_{max}	=	maximum stress
σ_{min}	=	minimum stress
$\Delta\sigma$	=	stress range
$\Delta\sigma_c$	=	critical stress range
$\Delta\sigma_{eff}$	=	effective stress range

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